Abstract

In high dimensional percolation at parameter $p < p_c$, the one-arm probability $\pi_p(n)$ is known to decay exponentially on scale $(p_c - p)^{-1/2}$. We show upper and lower bounds on the same exponential scale for the ratio $\pi_p(n)/\pi_{p_c}(n)$, establishing a form of a hypothesis of scaling theory. As part of our study, we provide sharp estimates (with matching upper and lower bounds) for several quantities of interest at the critical probability p_c. These include the tail behavior of volumes of, and chemical distances within, spanning clusters, along with the scaling of the two-point function at “mesoscopic distance” from the boundary of half-spaces. As a corollary, we obtain the tightness of the number of spanning clusters of a diameter n box on scale n^{d-6}; this result complements a lower bound of Aizenman (Nucl Phys B 485(3):551–582, 1997).